Relativistic formulation of quantum state diffusion?

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Abstract

The recently reported relativistic formulation of the well-known non-relativistic quantum state diffusion is seriously mistaken. It predicts, for instance, inconsistent measurement outcomes for the same system when seen by two different inertial observers.

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Introduction. Breuer and Petruccione (BP) [1] have quite recently reported the construction of a relativistic generalization of the stochastic Ito-Schrödinger-equations now widely used for open quantum systems. I have found that the model is completely mistaken. It can not reproduce even the ordinary quantum state diffusion theory as its non-relativistic limit. Its non-relativistic limit predicts, for instance, inconsistent experimental results for slowly moving observers. The reader might ignore my own interpretation of BP's *Concept* and might read directly the *Counter-example*.

The Concept. In Dirac's electron theory, to each space-like hyper-plane σ of the Minkowski-space a quantum state ψ is attributed. Such ψ is interpreted as the quantum state which is seen by the inertial observer residing on the hyper-plane σ. Let, for concreteness, the hyper-planes be parameterized by their unit normal vectors n and by their distance a from the origin. If we vary the hyper-surface, the state vector $\psi(\sigma) \equiv \psi(n,a)$ transforms unitarily:

$$d\psi = -iaH\psi - idn^\mu K_\mu \psi \quad , \tag{1}$$

where the Hamiltonian H and the boost operator K_μ depend also on the hyper-plane σ. The Eq.(1) can be split into two unitary equations:

$$\frac{\partial \psi}{\partial a} = -iH\psi \quad , \tag{2}$$

$$(\delta_\mu^\nu - n_\mu n^\nu)\frac{\partial \psi}{\partial n^\nu} = -iK_\mu \psi \quad . \tag{3}$$

This is standard Dirac theory so far [3]. BP will retain the second unitary equation (3) while replace the first equation (2) by the non-unitary Ito-Schrödinger equation of standard quantum state diffusion [2]:

$$\frac{\partial \psi}{\partial a} = -iH\psi + nonlinear, stochastic\ terms. \tag{4}$$

BP also present equations for the average state ρ, derived from Eqs.(3,4):

$$(\delta_\mu^\nu - n_\mu n^\nu)\frac{\partial \rho}{\partial n^\nu} = -i[K_\mu, \rho] \quad , \tag{5}$$

$$\frac{\partial \rho}{\partial a} = -i[H,\rho] + L\rho L^\dagger - \frac{1}{2}L^\dagger L\rho - \frac{1}{2}\rho L^\dagger L \quad . \tag{6}$$

The first equation is unitary, the second is not. BP claim to prove that their equations (3,4) as well as (5,6) are compatible, preserve Lorentz-invariance, and their translation invariance can also be pointed out in a restricted sense

[4]. Unfortunately, BP are unaware of further basic features of Dirac's theory. In particular, Dirac theory assures that various inertial observers have consistent experimental results. The BP equations fail to do so!

Assume, for instance, that two space-like hyper-planes σ_1 and σ_2 intersect at space time point x. Let A be a local scalar observable at x. Then in Dirac's theory the observable A transforms between σ_1 and σ_2 in such a way that its expectation value will not change:

$$\langle A\rangle_{\sigma_1} = \langle A\rangle_{\sigma_2} \quad , \tag{7}$$

where $\langle ...\rangle_\sigma$ stands for expectation values in quantum states ψ (or ρ, in general) taken on the hyper-plane σ. The claimed compatibility and relativistic invariance of the BP equations (3,4) and (5,6) are, in themselves, irrelevant since the physical consistency (7) of Dirac's theory is lost. How fatally it is lost the reader shall understand on a non-relativistic application.

The Counter-example. We do not even need BP's stochastic equations but the ensemble averaged ones (64,65) [cf. my Eqs.(5,6)], together with Eq.(63) which relates measured expectation values to the density operator (in the standard way, this time). Let us apply these equations to a typical non-relativistic situation. Consider a free non-relativistic electron in the authors' reference system $n_0 = (1, 0, 0, 0)$. We call the observer who rests in this reference system *R-observer.* Soon we need a moving *M-observer*, too, who rests in the reference system $n_v = (1, v/c, 0, 0)$ moving with a *small* non-relativistic velocity v with respect to the R-observer. The electron is non-relativistic for both observers. Assume it remains *localized* along the right x-axis at $x \approx \ell$. Both observers will measure the same spin-observable:

$$A = |\uparrow\rangle\langle\downarrow| + H.C. \quad . \tag{8}$$

They will measure *in coincidence*! For instance, the M-observer switches on his apparatus at time $a = 0$, in coincidence with the R-observer's apparatus at (his/her) time $a_0 = \ell v/c^2$. We expect that the two measurements lead to the same result in the non-relativistic limit $v/c \to 0$:

$$tr\{A\rho(n_v, 0)\} = tr\{A\rho(n_0, a_0)\} + \mathcal{O}(v/c) \quad . \tag{9}$$

Note that we can choose an arbitrary large distance ℓ so that $a_0 = \ell v/c^2$ remains relevant (e.g. constant) even for $v/c \to 0$.

For the electron's initial state in the frame of the R-observer we choose a superposition of the spin-up spin-down states:

$$\rho(n_0, 0) = \frac{1}{2}\Big(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|\Big) \quad . \tag{10}$$

(Observe that, initially, the rest observer would measure 1 for the expectation value of A.) We define the following Lindblad generator:

$$L(n_0,a) \equiv \frac{1}{\tau}|\uparrow\rangle\langle\uparrow| \quad . \tag{11}$$

Let the reduction time τ, controlling the strength of the quantum state diffusion, satisfy the condition $\tau << a_0$. Then the Eq.(64) turns the initial pure state (10) into the mixed one:

$$\rho(n_0,a_0) \approx \frac{1}{2}\Big(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\Big). \tag{12}$$

For time $a_0 = \ell v/c^2$, the quantum state has become reduced to the mixture of $|\uparrow\rangle$ and $|\downarrow\rangle$. The expectation value of the Hermitian observable A becomes zero for the the R-observer's measurement.

Let's turn to the M-observer. According to the Eq.(65), he/she initially sees the state

$$\rho(n_v,0) = \rho(n_0,0) + \mathcal{O}(v/c) \quad , \tag{13}$$

which is identical to the rest observer's initial state up to terms of the order of v/c. So, the M-observer measures 1 up to terms $\mathcal{O}(v/c)$.

In summary, we can write:

$$\langle A\rangle_{(n_v,0)} - \langle A\rangle_{(n_0,a_0)} = 1 + \mathcal{O}(v/c) \tag{14}$$

which, indeed, contradicts [5] to the invariance condition (7) of the Dirac theory.

Conclusion. Abandoning the standard Dirac wave functions has not been the main reason for BP's failure. The fatal reason is that relativistic Wiener-processes do not exist but trivial ones, as is particularly shown in the context of continuous wave function reduction theories by, e.g., Pearle [6] and myself [7]. Relativistic models of continuous reduction theories should first relax the Markovian approximation, cf. [8].

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References

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[3] In standard Dirac theory, H and K_μ have well-known expressions. BP are, for unclear reasons, troubling with their own awkward construction. Their wave function (11) [or (21)] is not Dirac's one! In BP's deliberate representation the Hamiltonian H and the boost K_μ have nothing to do with the well-known forms. One has no guess why BP depart from Dirac's standard wave functions.

[4] That translation invariance would be broken by the author's dynamic equations we demonstrate easily. For instance, let the parameters of three space-like hyper-planes $\sigma_1, \sigma_2, \sigma_3$ be, in obvious notations, chosen as follows: $n_1 = n_2 = (1,0,0,0)$, $n_3 = (2/\sqrt{3}, 0, 0, 1/\sqrt{3})$, $a_1 = 1$, $a_2 = 2$, $a_3 = 2/\sqrt{3}$. Let the two other reference frames R_1, R_2 have $(0,0,0,\sqrt{3}-2)$ and $(0,0,0,2\sqrt{3}-2)$ as their shifted origins. Elementary calculations show that $a_1 = a_3$ in R_1 and $a_2 = a_3$ in R_2, while n_1, n_2, n_3 do not change. Consider the averaged states ρ_1, ρ_2, ρ_3 respectively on the three hyper-planes. We apply Eq.(65) of BP in frame R_1 to relate ρ_1 and ρ_3: they turn out to be unitary equivalent. Similarly, we apply Eq.(65) in frame R_2 to ρ_2 and ρ_3: they, two, will be unitary equivalent. This implies that ρ_1 and ρ_2 must be unitary equivalent. But these latters are related non-unitarily by Eq.(64) (no matter which inertial frame is chosen).

[5] ℓ or $1/\tau$ (or both) diverge in the limit $v/c \to 0$. This does however not invalidate our Counter-example. First, it makes sense to assign any finite values to ℓ and $1/\tau$. Second, the violation of Eq.(7) by Eq.(14) is obvious already at finite v/c, i.e. at finite ℓ and $1/\tau$.

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